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Advanced Statistical Physics - Problem set 12

Summer Terms 2022

Hand in: Hand in tasks marked with * to mailbox no. (43) inside ITP room 105b by Friday 1.07. at 9:15 am.

19. Specific heat exponent and scaling relation *

4 Points

Calculate the specific heat critical exponent using

$$C_{\text{sing}}(t,h) = -T\frac{\partial^2}{\partial T^2} f_{\text{sing}}(t,h) \ ,$$

and the scaling hypothesis for $f_{\text{sing}}(t,h)$. Start from the generalized homogeneity equation

$$\lambda f_{\text{sing}}(t,h) = f_{\text{sing}}(\lambda^{a_t} t, \lambda^{a_h} h) \ .$$

Hint: Use an appropriate expression for λ to obtain the form of the singular part of the free energy as given in the lectures

$$f_{\text{sing}}(t,h) = |t|^c g_{f,\pm}(h/|t|^\Delta) \ .$$

Use the scaling of C_{sing} to relate c and α.

20. Coupled scalars *

1+3+2+2+2 Points

Consider the Hamiltonian

$$\beta\mathcal{H} = \int d^dx \left[\frac{t}{2}m^2 + \frac{K}{2}(\nabla m)^2 - hm + \frac{L}{2}(\nabla^2\phi)^2 + v(\nabla m)(\nabla\phi)\right] ,$$

coupling two one-component fields m and ϕ.

a) Write $\beta\mathcal{H}$ in terms of the Fourier transforms $m(\boldsymbol{q})$ and $\phi(\boldsymbol{q})$.

b) Construct a renormalization group transformation by rescaling distances such that $\boldsymbol{q}' = b\boldsymbol{q}$, and the fields such that $m'(\boldsymbol{q}') = \tilde{m}(\boldsymbol{q})/z$ and $\phi'(\boldsymbol{q}') = \tilde{\phi}(\boldsymbol{q})/y$. You do not need to evaluate the integrals that just contribute a constant additive term.

c) There is a fixed point such that $K' = K$ and $L' = L$. Find y_t, y_h and y_v at this fixed point.

d) The singular part of the free energy has a scaling form

$$f(t,h,v) = t^{2-\alpha} g(h/t^\Delta, v/t^\omega)$$

for t, h, v close to zero. Find α, Δ and ω.

e) There is another fixed point such that $t' = t$ and $L' = L$. What are the relevant operators at this fixed point, and how do they scale?